

RAMP STEER MANEUVER: TRANSIENT EFFECTS ON STEERING WHEEL ANGLE AND BODY SIDESLIP

Andrea Patrucco^{a,*}

^aVehicle Dynamics Engineer, Applus+ IDIADA

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The author uses the simplified approach to transients presented by Milliken&Milliken [1], based on a linearized bicycle model, to estimate the error on SWA/Ay and BSS/Ay curves due to a steady-state assumption being applied for a ramp steer maneuver. Some typical phenomena observed in plots commonly used to characterize a vehicle's behaviour in steady-state conditions are explained with fundamental parameters of the vehicles; both an ODE-based and a further simplified algebraic approach to quantify differences between ramp steer and steady state behavior are proposed.

INTRODUCTION

Ramp steer is a very common maneuver methodology to characterize a vehicle's behavior in pseudo-steady state conditions. In its most widespread formulation, the maneuver consists in letting a vehicle run at a constant velocity while increasing the steering wheel angle at a constant rate, resulting in the "ramp" naming.

The underlying assumption needed to consider this maneuver steady-state, is that the effect of the changes in steering wheel angle to the observed quantities is negligible, which leads to the necessity of choosing a relatively low steering rate value. Other needs, however, may influence the selection of such value: most importantly, the availability of a test track large enough to accommodate the complete trajectory of the vehicle, and secondly, for CAE applications, the added computational cost for longer simulations.

Figures 1 and 2 show simulation results for the same (non-linear) bicycle model subjected to a ramp-steer maneuver. In a first case, the ramp to 40deg takes place at 0.4deg/s over the span of 100 seconds; in a second case, the pace is brought to 12.5deg/s. The effect of ramp speed is evident in both cases.

While it is trivial to understand that the rate value will inevitably affect some transient behavior - a concept which can be easily proven through testing or simulating a vehicle at different steering wheel rates - the very nature of such effects is not always well understood. The aim of this paper is to provide a compact algebraic formulation of the issue which, while limited in accuracy by the simpleness of the model, might help identifying affecting factors and their relative importance.

We will first go through the assumptions and mathematical procedure to obtain a similar model to that of [1] and [2], then we will derivate from such model a more compact matrix formulation that can be effectively used to obtain a rough estimate of observed "steering

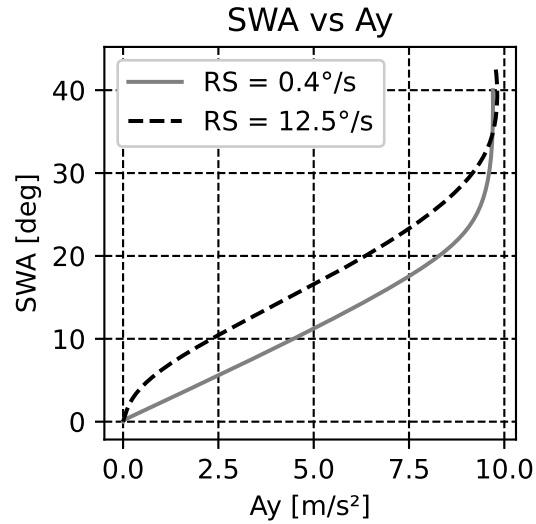


Figure 1. Non-linear bicycle model ramp-steer simulation: Steering wheel angle vs lateral acceleration

wheel angle offset" (with respect to true steady-state) in ramp-steer tests.

SYMBOLS

Through our formulation, we will refer to the following quantities:

- m : Vehicle mass
- J_z : Vehicle yaw inertia
- l : Vehicle wheelbase
- l_F : Distance between front axle and CoG
- l_R : Distance between rear axle and CoG
- CS_F : Front axle cornering stiffness
- CS_R : Rear axle cornering stiffness
- δ : Front axle steer angle

*to whom correspondence should be addressed

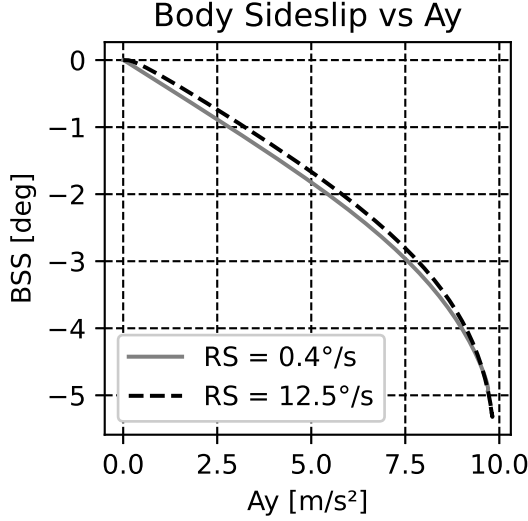


Figure 2. Non-linear bicycle model ramp-steer simulation: body sideslip vs lateral acceleration

- SWA: Steering wheel angle
- SR: Steering ratio (SWA/ δ)
- α_F : Front axle slip angle
- α_R : Rear axle slip angle
- β : Body sideslip angle
- $\dot{\beta}$: Body sideslip rate
- $\ddot{\beta}$: Body sideslip acceleration
- σ : Body yaw angle
- $\dot{\sigma}$: Body yaw rate
- $\ddot{\sigma}$: Body yaw acceleration
- V_X : Longitudinal speed in vehicle reference frame
- a_Y : Lateral acceleration in vehicle reference frame

LINEARIZED BICYCLE MODEL

We will base our approach on a linearized bicycle model. In this variant of the model, we approximate all trigonometric quantities as in Equation (1).

$$\cos(x) \approx 1; \sin(x) \approx x \quad (1)$$

We will also rely upon a linear tyre (in this case, axle) model, in which the bond between slip angle and lateral force will be described by Equation (2).

$$F_{y,i} = CS_i * \alpha_i \quad (2)$$

Since we are neglecting the effect of the front wheel rotation on front lateral force projection, and without considering longitudinal weight transfer phenomena (since we are assuming β to be reasonably small), the

only equations describing the state of the vehicle are two force balances: lateral force, moment around the Z axis, as seen in Equation (3).

$$\begin{cases} J_Z * \sigma'' = CS_F l_F \alpha_F - CS_R l_R \alpha_R \\ m * a_Y = CS_F \alpha_F + CS_R \alpha_R \end{cases} \quad (3)$$

In normal circumstances, when we are willing to describe the linearized behavior of the vehicle in steady-state conditions, we can assume $\sigma'' = 0$, which leads us to a system of two equations in two unknowns, α_F and α_R .

Furthermore, exploiting the (linearized) geometrical relations that bind, on one hand, sideslip and front axle steer angle and, on the other hand, corner geometry, longitudinal speed, lateral acceleration and slip angles, we obtain, under the steady-state assumption, Equation (4).

$$\begin{cases} \beta_{SS} = -\alpha_R + \frac{a_Y l_R}{V_X^2} \\ \delta_{SS} = -\alpha_R + \alpha_F + \frac{a_Y l}{V_X^2} \end{cases} \quad (4)$$

Which in turns brings us to the following, steady state, linearized Equation (5) for body sideslip and front axle steered angle:

$$\begin{cases} \beta_{SS} = \left(\frac{l_R}{V_X^2} - \frac{m * l_F}{CS_R} \right) * a_Y \\ \delta_{SS} = \left(\frac{l}{V_X^2} + \frac{m * l_R}{CS_F} - \frac{m * l_F}{CS_R} \right) * a_Y \end{cases} \quad (5)$$

Note that Equation (4) and Equation (5) have been written by assuming terms V_X^2 and a_Y are controlled, as in the case of a "steady-state, constant velocity" condition, where we hypothetically let lateral acceleration increase while longitudinal speed is kept constant in order to observe effects on the front wheel steered angle and sideslip.

TRANSIENT BEHAVIOR

Transient behavior of the linearized bicycle model can be still described in Equation (3), which binds inertia forces to slip angles. Our goal is to obtain an algebraic form equivalent to that of the simplified formulae in Equation (4) to still obtain an estimation of β and δ : a first step will hence be that of explicating the relation between these quantities and slip angles. By assuming that the vehicle body is moving with speeds (relative to itself) of V_X and V_Y in longitudinal and lateral direction respectively, and knowing its yaw rate σ' , we can write Equation (6), which can be linearized and rearranged into Equation (6).

$$\begin{cases} \beta = \arctan(\frac{V_Y}{V_X}) \\ \alpha_F = \delta - \arctan(\frac{V_Y + l_F \sigma'}{V_X}) \\ \alpha_R = -\arctan(\frac{V_Y - l_R \sigma'}{V_X}) \end{cases} \quad (6)$$

$$\theta' = \frac{a_Y}{V_X} \quad (12)$$

$$\sigma' = \frac{a_Y}{V_X} - \beta' \quad (13)$$

$$\begin{cases} \alpha_F = \delta - \beta - \frac{l_F \sigma'}{V_X} \\ \alpha_R = -\beta + \frac{l_R \sigma'}{V_X} \end{cases} \quad (7)$$

$$\sigma'' = \frac{a'_Y}{V_X} - \beta'' \quad (14)$$

It is then possible to replace slip angles in Equation (3) with their linearized estimation in Equation (7), leading to Equation (8).

With Equation (13) and Equation (14), we can remove all σ terms from Equation (8), transforming it into Equation (15).

$$\begin{cases} J_Z \sigma'' = C_{S_F} l_F \delta + (C_{S_R} l_R - C_{S_F} l_F) \beta - \\ -(C_{S_R} \frac{l_R^2}{V_X} + C_{S_F} \frac{l_F^2}{V_X}) \sigma' \\ m a_Y = C_{S_F} \delta - (C_{S_F} + C_{S_R}) \beta + \\ +(C_{S_R} \frac{l_R}{V_X} - C_{S_F} \frac{l_F}{V_X}) \sigma' \end{cases} \quad (8)$$

$$\begin{cases} J_Z (\frac{a'_Y}{V_X} - \beta'') = C_{S_F} l_F \delta - D_1 \beta - \frac{S_2}{V_X^2} a_Y + \frac{S_2}{V_X} \beta' \\ (m + \frac{D_1}{V_X}) a_Y = C_{S_F} \delta - S_0 \beta + \frac{D_1}{V_X} \beta' \end{cases} \quad (15)$$

Where, for compactness, we defined the quantities in Equation (16), Equation (17) and Equation (18).

$$D_1 = C_{S_F} l_F - C_{S_R} l_R \quad (16)$$

$$S_2 = C_{S_F} l_F^2 + C_{S_R} l_R^2 \quad (17)$$

$$S_0 = C_{S_F} + C_{S_R} \quad (18)$$

The system of equations in Equation (8) still has four unknowns: σ' , σ'' , β and δ . In order to replace the σ terms with something closer to our variables of interest, we will take advantage of the relations between V_X and a_Y , by assuming the motion of the vehicle can be described as a rigid motion around an instantaneous rotation center located at a distance denoted with R , rotating with a speed θ' . Equation (9) and Equation (10) describe the motion of the vehicle, reprojected in the vehicle's reference system.

$$V_{c,c} = \begin{bmatrix} V_X \\ V_Y \end{bmatrix} = \begin{bmatrix} R \theta' \cos(\beta) \\ R \theta' \sin(\beta) \end{bmatrix} \quad (9)$$

$$a_{c,c} = \begin{bmatrix} a_X \\ a_Y \end{bmatrix} = \begin{bmatrix} -R \theta'^2 \sin(\beta) \\ R \theta'^2 \cos(\beta) \end{bmatrix} \quad (10)$$

It is worth noting that, even though the vehicle's center of mass is following the trajectory described by Equation (9), yaw rate and θ' are not coupled, that is, the angle difference between the virtual radius connecting the CoG to the center of rotation and the direction of the vehicle (β plus a constant) will change in time, leading to the relation expressed in Equation (11).

$$\sigma' = \theta' - \beta' \quad (11)$$

Furthermore, by comparison between $a_{c,c}$ and $V_{c,c}$ we can identify the relation in Equation (12), which allows us to replace θ' in Equation (11) leading to Equation (13). Time-derivation leads to Equation (14).

The way we take advantage of these equations depends on the approach that we decide to follow, that is, either numerically solve a differential equation, or, through some simplifying assumptions, look for an algebraic solution.

DIFFERENTIAL EQUATION APPROACH

The system described by Equation (15) still presents some issues that prevent its numerical solution under an ODE form: most notably, only the first equation contains both a_Y and β in its first and second derivative form, respectively. To solve this, we can replace the second equation with its time-derivative, leading to Equation (19).

$$\begin{cases} J_Z (\frac{a'_Y}{V_X} - \beta'') = C_{S_F} l_F \delta - D_1 \beta - \frac{S_2}{V_X^2} a_Y + \frac{S_2}{V_X} \beta' \\ (m + \frac{D_1}{V_X}) a'_Y = C_{S_F} \delta' - S_0 \beta' + \frac{D_1}{V_X} \beta'' \end{cases} \quad (19)$$

This set of equations can be represented in a more familiar form in Equation (20).

$$[M]\vec{q}'' = [R]\vec{q}' + [K]\vec{q} + \vec{u} \quad (20)$$

Where matrices M, R and Q are described in Equation (21), Equation (22), Equation (23) and Equation (24). The column status vector $\vec{q}$ and its time derivatives are defined in Equation (25).

$$M = \begin{bmatrix} \frac{J_Z}{V_X} & -J_Z \\ (m + \frac{D_1}{V_X}) & -\frac{D_1}{V_X} \end{bmatrix} \quad (21)$$

$$R = \begin{bmatrix} -\frac{S_2}{V_X^2} & \frac{S_2}{V_X} \\ 0 & -S_0 \end{bmatrix} \quad (22)$$

$$K = \begin{bmatrix} 0 & -D_1 \\ 0 & 0 \end{bmatrix} \quad (23)$$

$$\vec{u} = \begin{bmatrix} CS_F l_F \delta \\ CS_F \delta' \end{bmatrix} \quad (24)$$

$$\vec{q} = \begin{bmatrix} \int a_Y \\ \beta \end{bmatrix}; \vec{q}' = \begin{bmatrix} a_Y \\ \beta' \end{bmatrix}; \vec{q}'' = \begin{bmatrix} a_Y' \\ \beta'' \end{bmatrix} \quad (25)$$

By imposing a constant longitudinal speed V_X , and setting initial conditions of zero for both lateral acceleration a_Y , sideslip β and its derivative β' , we can use a numerical integration method such as Euler's to obtain time-histories for lateral acceleration and body sideslip, as shown in the Results section.

Note that this system can (and should) be further simplified to a single degree of freedom as in [1]. Such representation bears obvious advantages compared to the one presented here, whose usefulness is limited to the derivation of equations in the following section.

ALGEBRAIC APPROACH

Even though the differential equation approach, presented in Equation (20) has a rather low computational cost, it still requires time-integration techniques and further data processing (for example, to extrapolate a SWA/ a_Y curve) in order to obtain useful information. For this reason, one might want to try and integrate Equation (15) with some further simplifying assumptions. As we will see in the Results section, trends of both body sideslip and lateral acceleration tend to assume the form of a line, when plotted against time, after an initial oscillation. This suggests that, if we are only interested in behavior after the start of the ramp steer maneuver, we can assume $\beta'' = 0$ and $a_Y'' = 0$.

We can see this approach as a sort of "step forward" compared to the full steady state hypothesis: while there we worked under the assumption of $\beta' = 0$ and $a_Y' = 0$, here we move slightly further, considering the effect of

the first time-derivatives, but neglecting the second.

To follow this approach, we will couple both expressions in Equation (15) with their time-derivatives under the aforementioned assumptions, obtaining Equation (26).

$$\begin{cases} J_Z(\frac{a_Y'}{V_X} - \beta'') = CS_F l_F \delta - D_1 \beta - \frac{S_2}{V_X^2} a_Y + \frac{S_2}{V_X} \beta' \\ (m + \frac{D_1}{V_X}) a_Y = CS_F \delta - S_0 \beta + \frac{D_1}{V_X} \beta' \\ 0 = CS_F l_F \delta' - D_1 \beta' - \frac{S_2}{V_X^2} a_Y' \\ (m + \frac{D_1}{V_X^2}) a_Y' = CS_F \delta' - S_0 \beta' \end{cases} \quad (26)$$

We assume that the rate of increase of steered front wheel angle δ' , as well as lateral acceleration a_Y , are known, and we look for a solution under the form Equation (27), where $\vec{x}$ and $\vec{y}$ are described in .

$$[C]\vec{x} = [V]\vec{y} \quad (27)$$

$$\vec{x} = \begin{bmatrix} \delta \\ \beta \\ a_Y' \\ \beta' \end{bmatrix} \quad (28)$$

$$\vec{y} = \begin{bmatrix} \delta' \\ a_Y \end{bmatrix} \quad (29)$$

By rearranging Equation (26) into matrix form, we obtain the expressions for $[C]$ and $[V]$ listed in Equation (30) and Equation (31).

$$C = \begin{bmatrix} -CS_F l_F & D_1 & \frac{J_Z}{V_X} & -\frac{S_2}{V_X} \\ -CS_F & S_0 & 0 & -\frac{D_1}{V_X} \\ 0 & 0 & (m + \frac{D_1}{V_X}) & S_0 \\ 0 & 0 & \frac{S_2}{V_X^2} & D_1 \end{bmatrix} \quad (30)$$

$$V = \begin{bmatrix} 0 & -\frac{S_2}{V_X^2} \\ 0 & -(m + \frac{D_1}{V_X}) \\ CS_F & 0 \\ CS_F l_F & 0 \end{bmatrix} \quad (31)$$

We can now solve Equation (27) with any linear algebra calculation tool. Results obtained with this method provide an adequate approximation of its ODE counterpart outside of the near-zero lateral acceleration region, where they tend to over-estimate the difference between steady-state and transient values for β and δ , as we will see in more detail in the Results section.

ZERO SIDESLIP RATE APPROACH

Coming back to Equation (3), it would be tempting to just mix them with the expressions in Equation (4), to find β and δ as functions of a_Y and σ'' . If we can provide an estimate for yaw acceleration, this approach is possible, however it should be noted that Equation (4) is only valid when $\beta' = 0$, which is in direct opposition to our main goal.

Still, we can use Equation (26) to estimate (under its same assumptions) both σ'' through a_Y' as in Equation (32).

$$\sigma'' = \frac{CS_F l_F l}{\frac{1}{V_X} CS_F CS_R l^2 - m V_X (CS_F l_F - CS_R l_R)} \quad (32)$$

Now, combining Equation (3) and Equation (4), we obtain the approximate expression for ramp steer δ and β in Equation (33).

$$\begin{cases} \beta \approx \beta_{SS} + \frac{J_Z \sigma''}{CS_R l} \\ \delta \approx \delta_{SS} + \frac{J_Z \sigma''}{l} \left(\frac{1}{CS_F} + \frac{1}{CS_R} \right) \end{cases} \quad (33)$$

While this form might be useful to obtain an intuitive idea of the effects of a positive yaw acceleration on a ramp steer maneuver results with respect to a truly steady-state case, results tend to always under-estimate the true difference, as we will see in the Results section.

RESULTS

Figures 3 and 4 depict a comparison between different estimations of steering wheel angle transient behaviour during a ramp steer event. The ramp speed is 13.5deg/s; vehicle parameters are not directly comparable to the example shown in figure 1.

Note that all models agree on the so-called steering wheel angle gradient, i.e., at a reasonable distance from the axes origin, the derivative of steering wheel angle with respect to a_Y is the same for all models. The algebraic approach (based on the hypothesis of $\beta'' = 0$ and $a_Y'' = 0$) is also accurate in estimating the correct height of the curve once it has settled (which, in this case, happens after $\approx 1 \frac{m}{s^2}$). The simplified formulae fail to fully account for the difference in SWA caused by transient effects. Body Sideslip (Figures 5 and 6) follows a similar pattern, with the ODE approach being able to depict the initial, positive sign of sideslip induced by the abrupt introduction of steered angle at the front wheel.

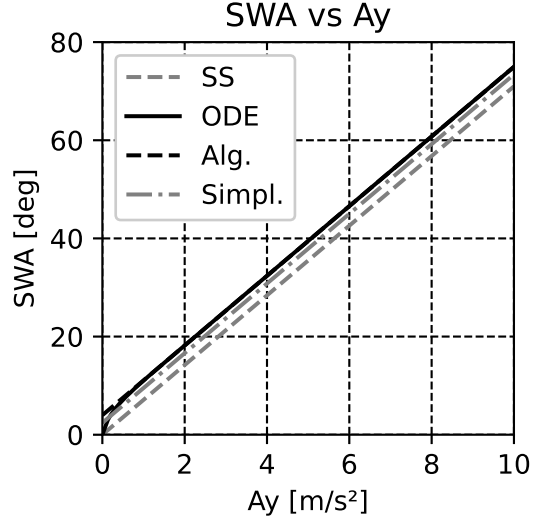


Figure 3. Comparison between proposed calculation methods for steering wheel angle estimation

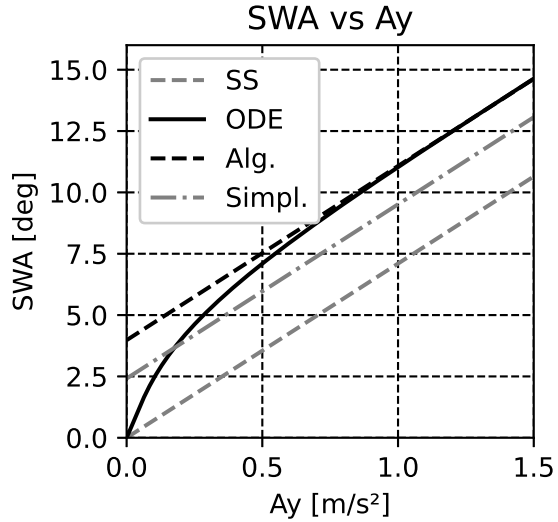


Figure 4. Comparison between proposed calculation methods for steering wheel angle estimation - Close-up

SENSITIVITY ANALYSIS

Since we have shown that the algebraic model is able to approximate the behavior of the ODE model far from the axes origin, it is interesting to use it to obtain an idea of the relative importance of the parameters on δ and β

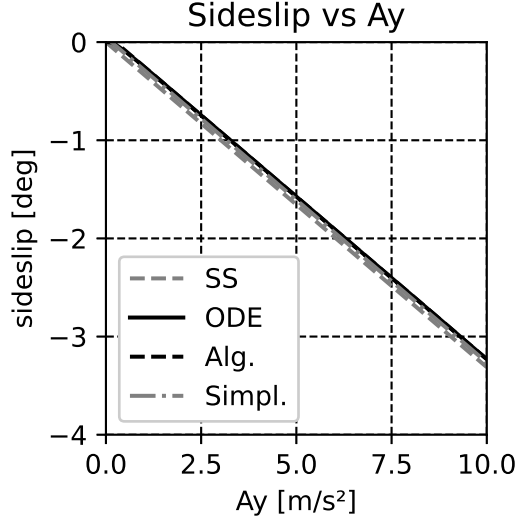


Figure 5. Comparison between proposed calculation methods for sideslip estimation

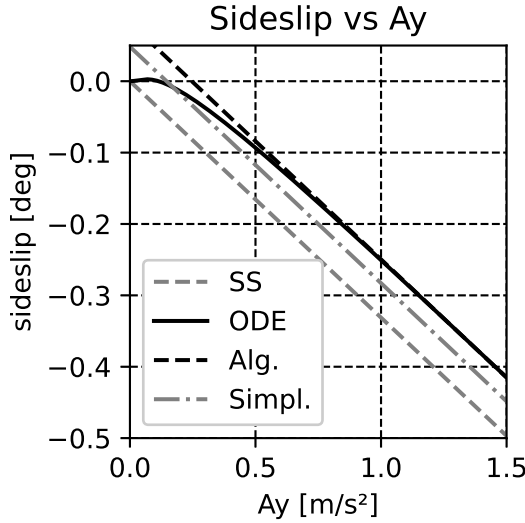


Figure 6. Comparison between proposed calculation methods for sideslip estimation - Close-up

compared to their steady-state counterpart. We propose the example vehicle whose data is resumed in table 1.

Table 2 shows the variations in results for positive and negative changes in the order of 10% on all parameters listed before. Results are shown considering a lateral acceleration of $4 \frac{m}{s^2}$.

Table 1. Sample vehicle parameters

Description	unit	value
mass	kg	1570
Yaw inertia	kgm ²	3218
Distance front axle - CoG	m	1.154
Distance rear axle - CoG	m	1.546
Steering Ratio	-	15
Front Axle Cornering Stiffness	N/deg	1984
Rear Axle Cornering Stiffness	N/deg	1694
Ramp rate	deg/s	13.5
Longitudinal Speed	km/h	100

Table 2. Sensitivity Analysis Results for $a_Y = 4 \frac{m}{s^2}$

Description	W_{SS}	W	ΔW	β_{SS}	β	$\Delta\beta$
Reference	15.45	19.34	3.89	-1.125	-0.982	0.143
m +10%	15.79	19.80	4.01	-1.283	-1.136	0.148
m -10%	15.11	18.88	3.78	-0.967	-0.828	0.139
J +10%	15.45	19.59	4.14	-1.125	-0.973	0.152
J -10%	15.45	19.10	3.65	-1.125	-0.991	0.134
SR +10%	16.99	20.89	3.89	-1.125	-0.995	0.130
SR -10%	13.90	17.80	3.89	-1.125	-0.966	0.159
rate +10%	15.45	19.73	4.28	-1.125	-0.967	0.158
rate -10%	15.45	18.95	3.51	-1.125	-0.996	0.129
CS_F +10%	12.98	17.49	4.51	-1.125	-0.955	0.171
CS_F -10%	18.47	21.83	3.36	-1.125	-1.005	0.120
CS_R +10%	17.61	20.76	3.15	-0.981	-0.869	0.112
CS_R -10%	12.81	17.96	5.15	-1.301	-1.106	0.195
l_F +135mm	10.30	16.57	6.28	-1.350	-1.128	0.223
l_F -135mm	20.60	23.31	2.71	-0.900	-0.798	0.102
V = 110km/h	13.36	17.56	4.20	-1.205	-1.050	0.155
V = 90km/h	18.27	21.80	3.53	-1.017	-0.888	0.130

It is immediately observed that whatever causes the vehicle to reach the same target lateral acceleration "faster" (i.e., whatever reduces the steady-state theoretical value for steering wheel angle) causes a comparatively larger error between ramp steer and steady-state conditions. This is particularly evident with dramatic changes in the CoG longitudinal position:

when moving the center of gravity 135mm forward vs backwards, in spite of the steady-state SWA value doubling, error on both SWA and sideslip is less than half than in the rear-biased case.

Another interesting effect is that the SWA delta is unaffected by changes in the steering ratio, in terms of absolute value: while the steady-state steering wheel angle needed to reach a given acceleration changes proportionally, the absolute SWA error remains constant, becoming comparatively more important for the more direct steering case. Similarly, when increasing or decreasing the ramp rate, there is a direct proportionality in the error. This suggests that there is a perfectly direct proportionality between the two quantities, at least in this approximated model. Note that body sideslip estimation and its error "feel" the equivalent effect of a change in ramp rate and in steering ratio exactly in the same way.

On the sideslip topic, we may see that the ramp steer maneuver transients introduce an influence of front axle characteristic on the absolute value of sideslip. Considering changes in front axle cornering stiffness, we can observe that a stiffer front axle is associated to an increased ramp-to-ss error, which leads to a reduced sideslip at the same lateral acceleration level.

CONCLUSION

The ramp steer maneuver, commonly used in the automotive industry as a proxy for steady state condition, presents, when executed at the rate mandated by the current ISO norms, significant differences with respect to a true steady-state condition.

Though sometimes attributed to elasto-kinematic properties or to tire behavior, transient effects on steering wheel angle and on body sideslip values can be justified by simple equations directly obtained from a linear bicycle model and rigid body kinematics formulae. A reasonably simple algebraic equation, based upon the hypothesis of null body sideslip acceleration and null yaw jerk, can be effectively used to bind the differences between steady-state and ramp-steer to basic geometric and mass properties of a vehicle, and to its axles' cornering stiffness. A further simplification proved not to be accurate enough to provide valuable results.

Transient effects are responsible for a significant over-estimation of steady-state steering wheel angle to obtain

a given lateral acceleration. This makes it tricky, for vehicles with a very direct steering system, to evaluate reference steering wheel angle values to achieve a given lateral acceleration (for example, to perform a step-steer maneuver). Effects are particularly interesting for such regulations as ESC certification ([3]), whose initial phase mandates a ramp steer maneuver at 80km/h to identify the SWA value at which a lateral acceleration of 0.3g is obtained: the over-estimation is likely to cause a non-negligible disadvantage for sportscars.

Not only transient effects cause higher (i.e., closer to zero) body sideslip angle values: they introduce, through yaw acceleration, a dependency between sideslip and front axle characteristics, even in a linear model.

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REFERENCES

1. W. Milliken and D. Milliken, Race Car Vehicle Dynamics, *SAE International*, 1995, ISBN1560915269
2. William A. Cobb, Engineering Report PG-61187, *GM*, 1993.
3. Regulation of the Economic Commission for Europe of the United Nations (UN/ECE) - Uniform Provisions concerning the approval of passenger cars with regard to Electronic Stability Control (ESC) Systems [2018/1592], *OK L 269 26.10.2018*.